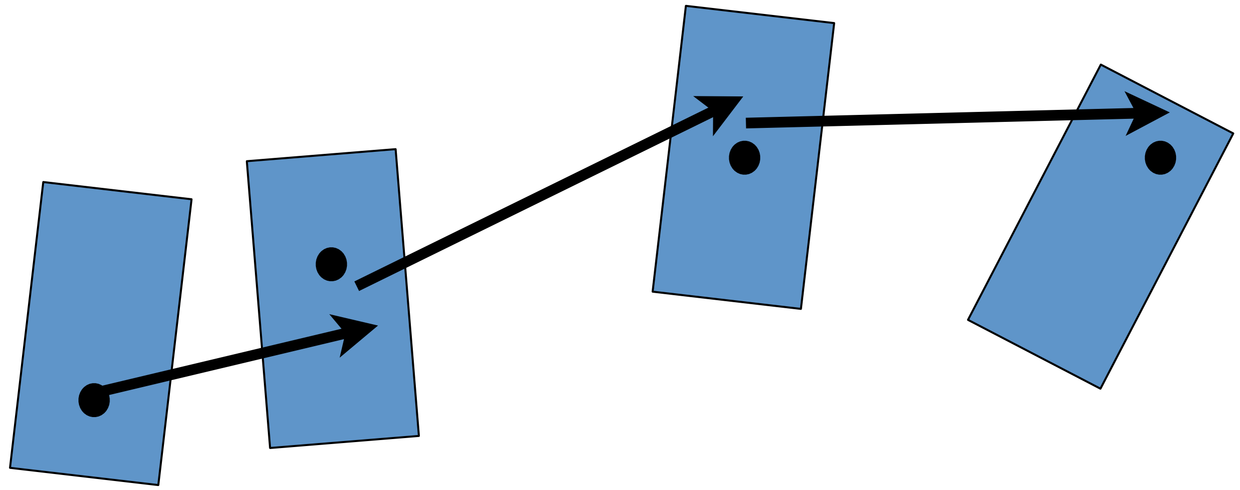
Kalman fi{l,t}ter update

Eric Church, 5-Apr-2012

This approach uses Spacepoints

- We rely on SpacepointServices to produce 3D spacepoints and their errors from sensible clusters.
- Then run a Kalman-filtered track through those points.

Kalman track fi{l,t}ter



$$\vec{x} = \begin{pmatrix} \frac{q}{p} \\ \frac{du}{dw} \\ \frac{dv}{dw} \\ u \\ v \end{pmatrix}$$

$$\theta \sim 1/p$$

LArSoft's KF implements all material effects, accommodates a magnetic field.

New effort: 3D tracks from 2D hits.

Kalman Filter

$$\vec{x_k} = \tilde{\vec{x_k}} + K_k \vec{r_k}$$

$$C_k = (I - K_k H_k)\tilde{C}_k$$

where

$$\vec{r_k} = \vec{m_k} - H_k \tilde{\vec{x_k}}$$

~x_k+1 is arrived at using Geant3 based track-following code. Not a transport matrix, as is common elsewhere.

The $\tilde{}$ denotes a predicted quantity. k labels a point on the track. x is the state space and m is the measurement. They are connected by H. The Kalman gain is

$$K_k = \tilde{C}_k H_k^T (H_k \tilde{C}_k H_k^T + V_k)^{-1}$$

Kalman tracking

- Working in uBooNE geometry
- Am following Icarus's inspiration: I measure not just, x,y,z, but also theta, ux, uy, uz. Do the latter by comparing point-to-point deflection over 2 and 3 spacepoints.
- Problem had been that ranging-out muons give back a good momentum value, but *uncontained muons* which rely on good measurement of Multiple Scattering gives back a random momentum value.
- I have implemented the full 5x5 solution. Have inserted the correct Jacobian that connects the local, planar (u,v) 5-element measurement to the detector coordinates --theta, ux, uy, uz, x, y, z-- to which they project.
- **Learned how to not segfault due to outlier theta corrections**: basically allow only inner 2sigma of Kalman updates, and limit p search range to 3 orders of magnitude.

Kalman tracking

- I’m in my own private release, pretty out of sync with $SRT_PUBLIC_CONTEXT. ...
- I’m using Spacepoints that now contain the new point-by-point 3x3 error matrix. (Thanks, Herb.) And still this has not exactly represented the breakthrough hoped for.

Measurements and Errors

$$u_x = \frac{(x_2 - x_1)^2}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 (z_2 - z_1)^2}}$$

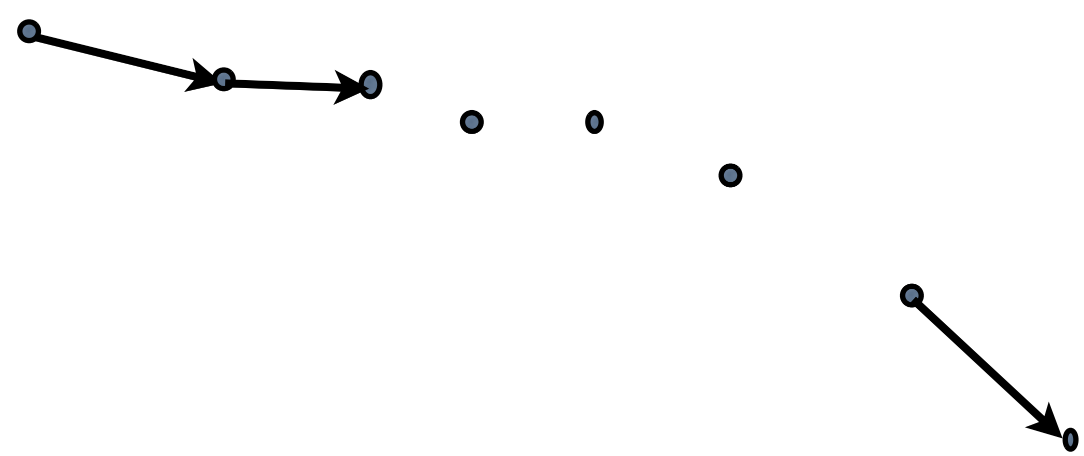
$$cos\theta = \vec{u}_i \cdot \vec{u}_{i-1}$$

From Herb's Spacepoints

$$\delta^2 u_x = \frac{\partial^2 u_x}{\partial^2 x_1} \delta^2 x_1 + ... + \frac{\partial^2 u_x}{\partial^2 z_2} \delta^2 z_2 + \frac{\partial^2 u_x}{\partial y_1 \partial z_1} \delta(y_1 z_1) + \frac{\partial^2 u_x}{\partial y_2 \partial z_2} \delta(y_2 z_2)$$

muons

The overall deflection describes a, say, 5 GeV/c muon. But for the fitter to get this answer it must know the overall situation while knowing at the same time that point to point the deflections are milliradianish: it is an intricate problem.



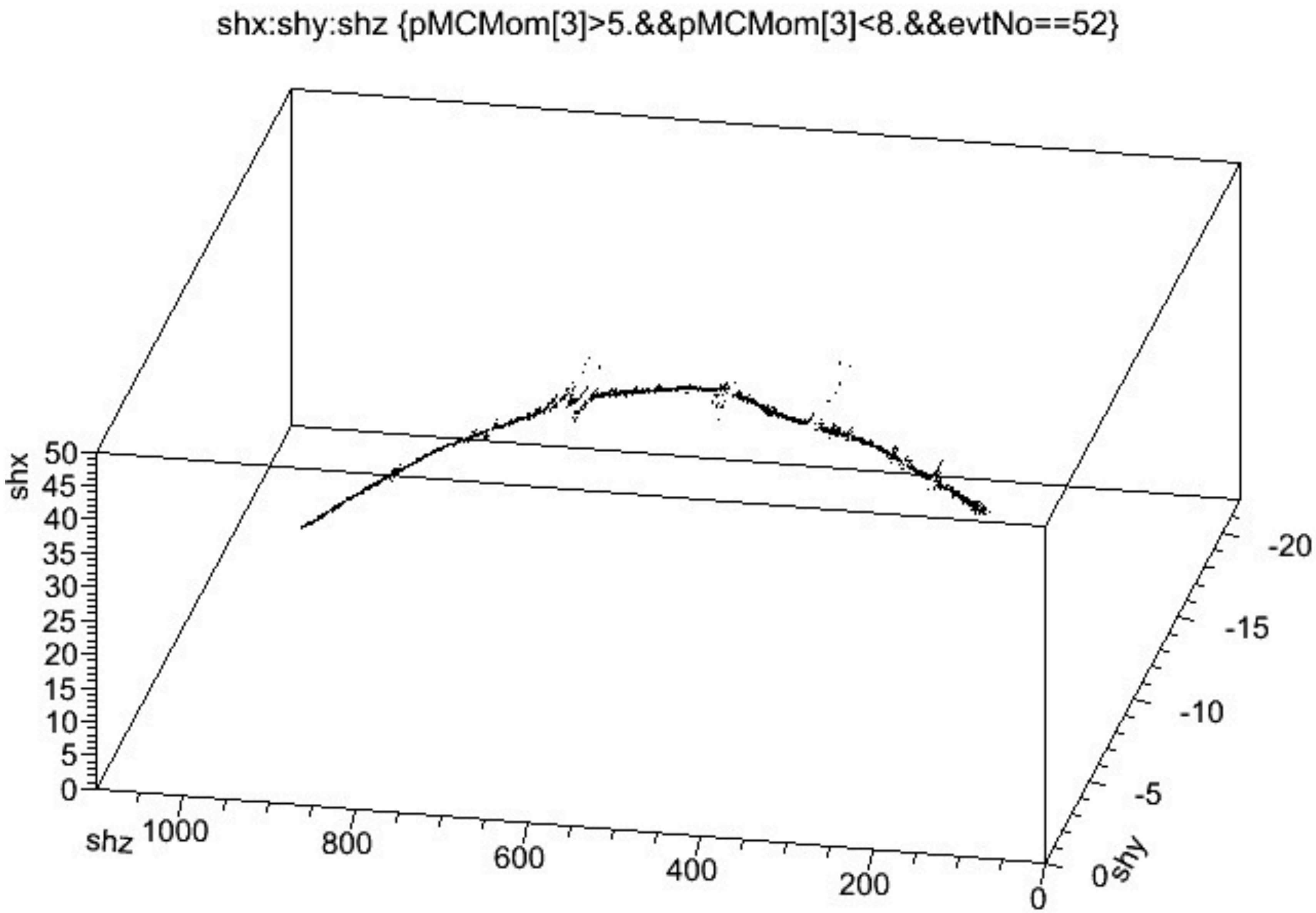
Nevertheless, if all the errors are correct it really ought to work.

...If we can convince ourselves that our theta resolution is reasonable.

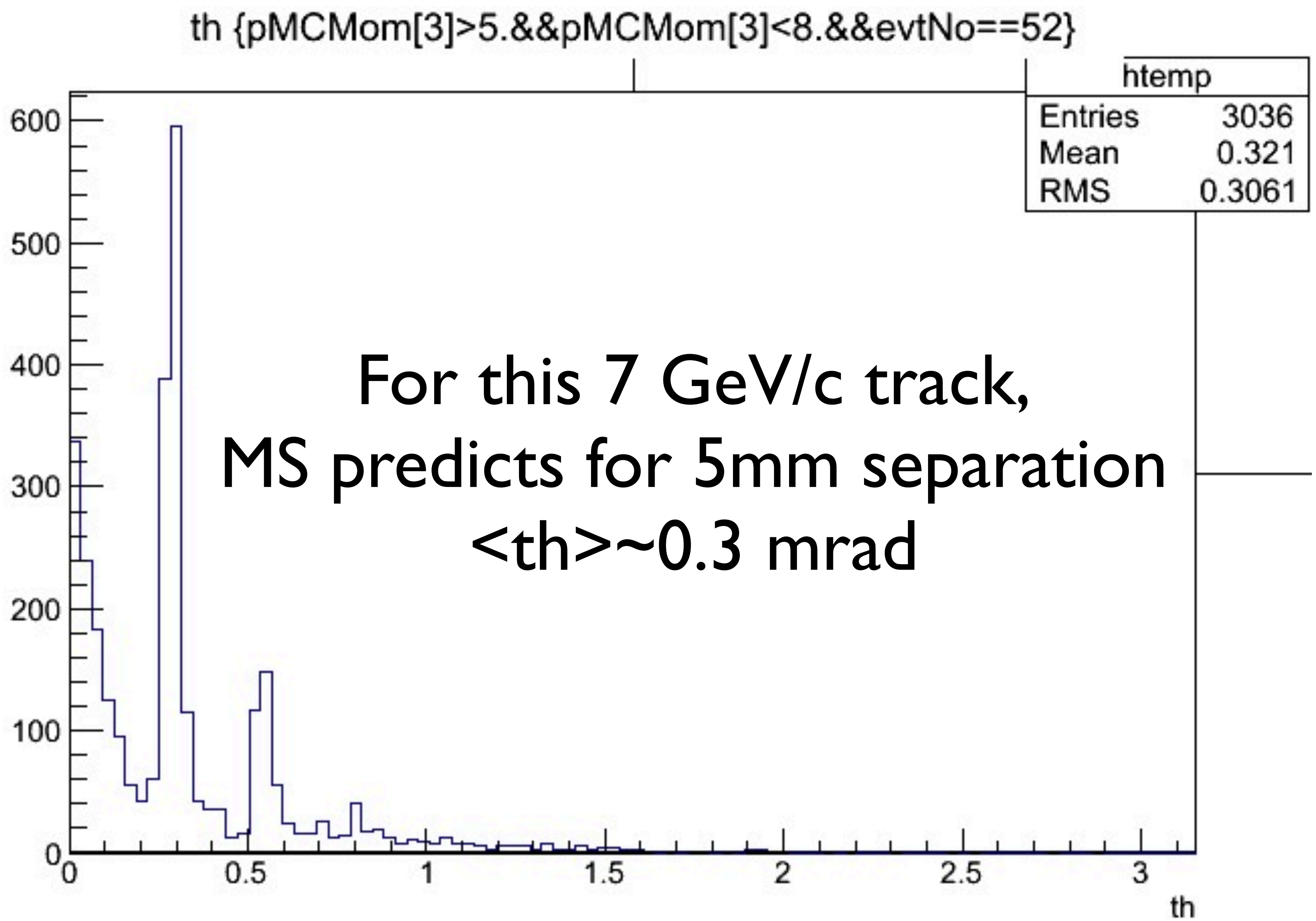
MC Muons

- I generate 0.5, 1.0, 1.5, 2.0, 2.5, 4.0, 7.0, 10.0 GeV/c MC muons in MicroBooNE, and reconstruct them through SpacePointFinder. Then I run my Track3DKalmanSPS on the output.

7 GeV/c (exiting mu)



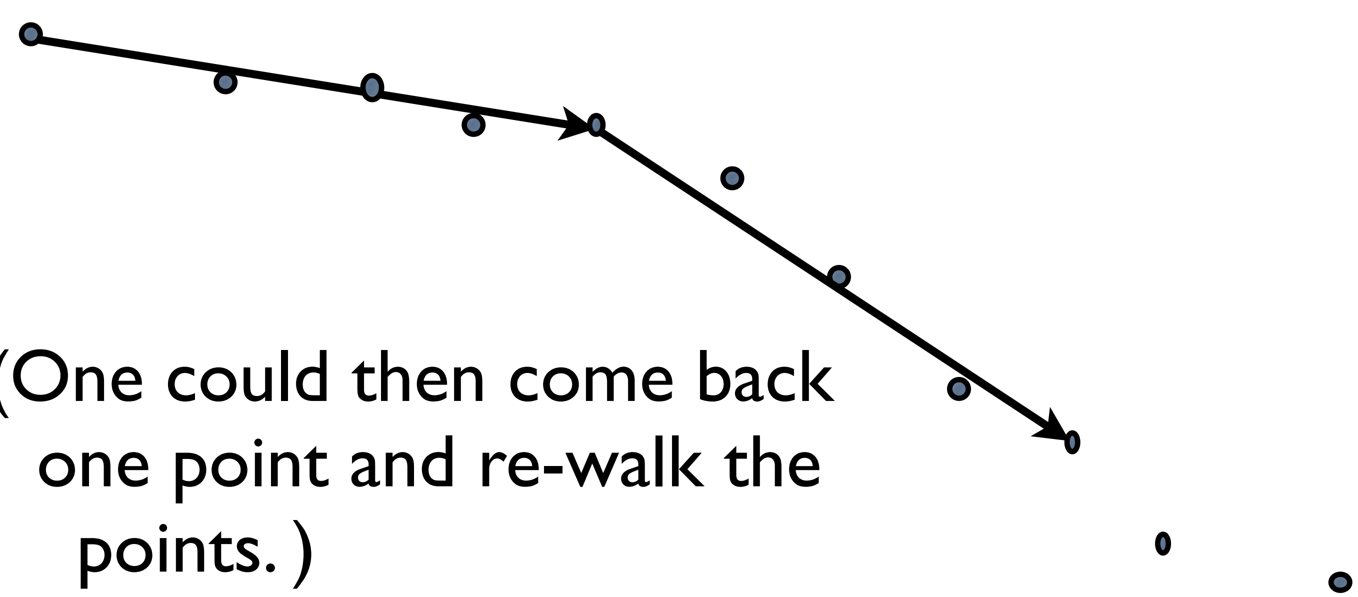
point-to-point scatter angle distribution



For this 7 GeV/c track,
MS predicts for 5mm separation
<th>~0.3 mrad

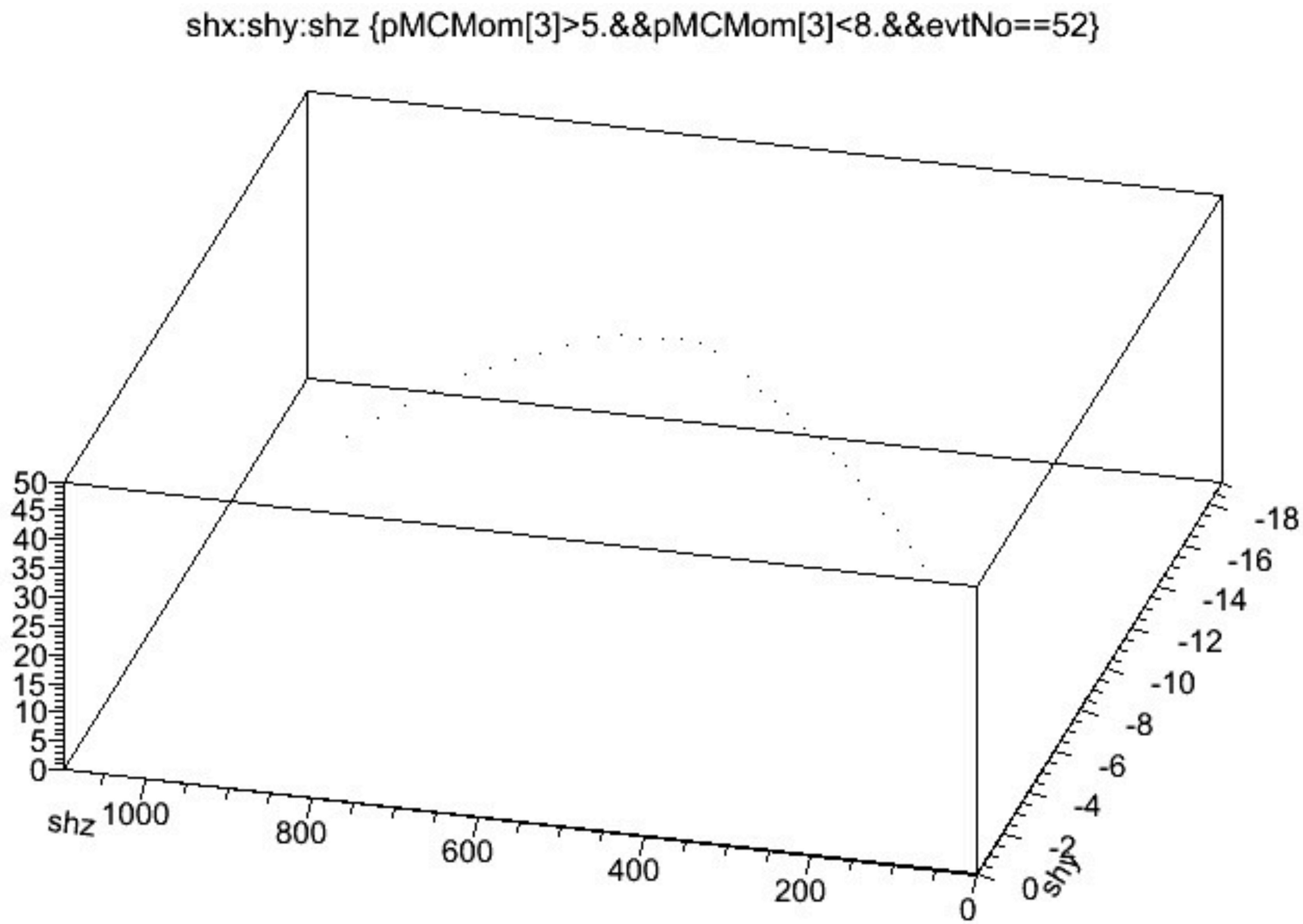
Using every Nth spacepoint

Icarus mentions the need to do the following

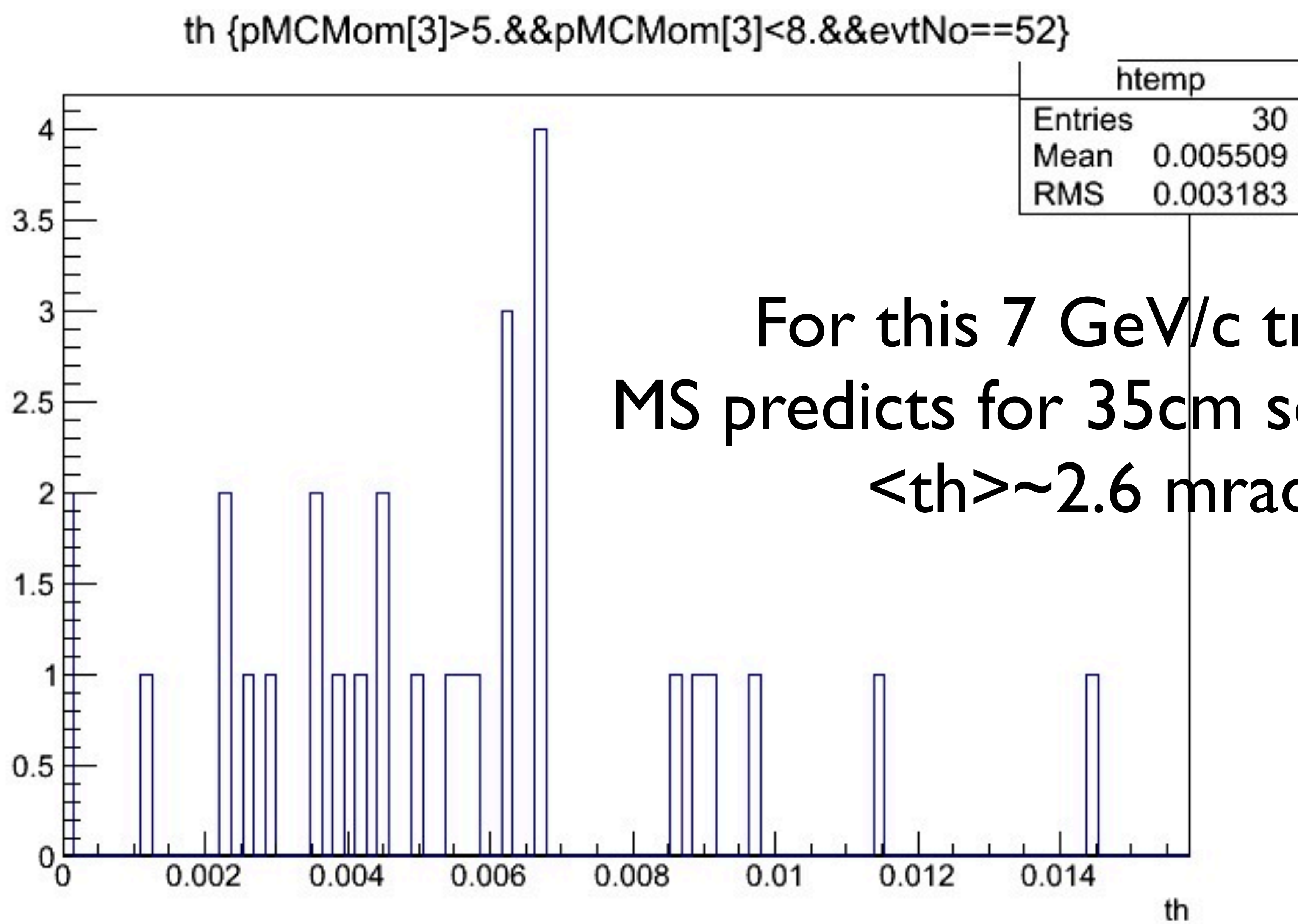


... decimation. (One could then come back and move fwd one point and re-walk the points.)

“Decimate” (Only every 100th SpacePoint)



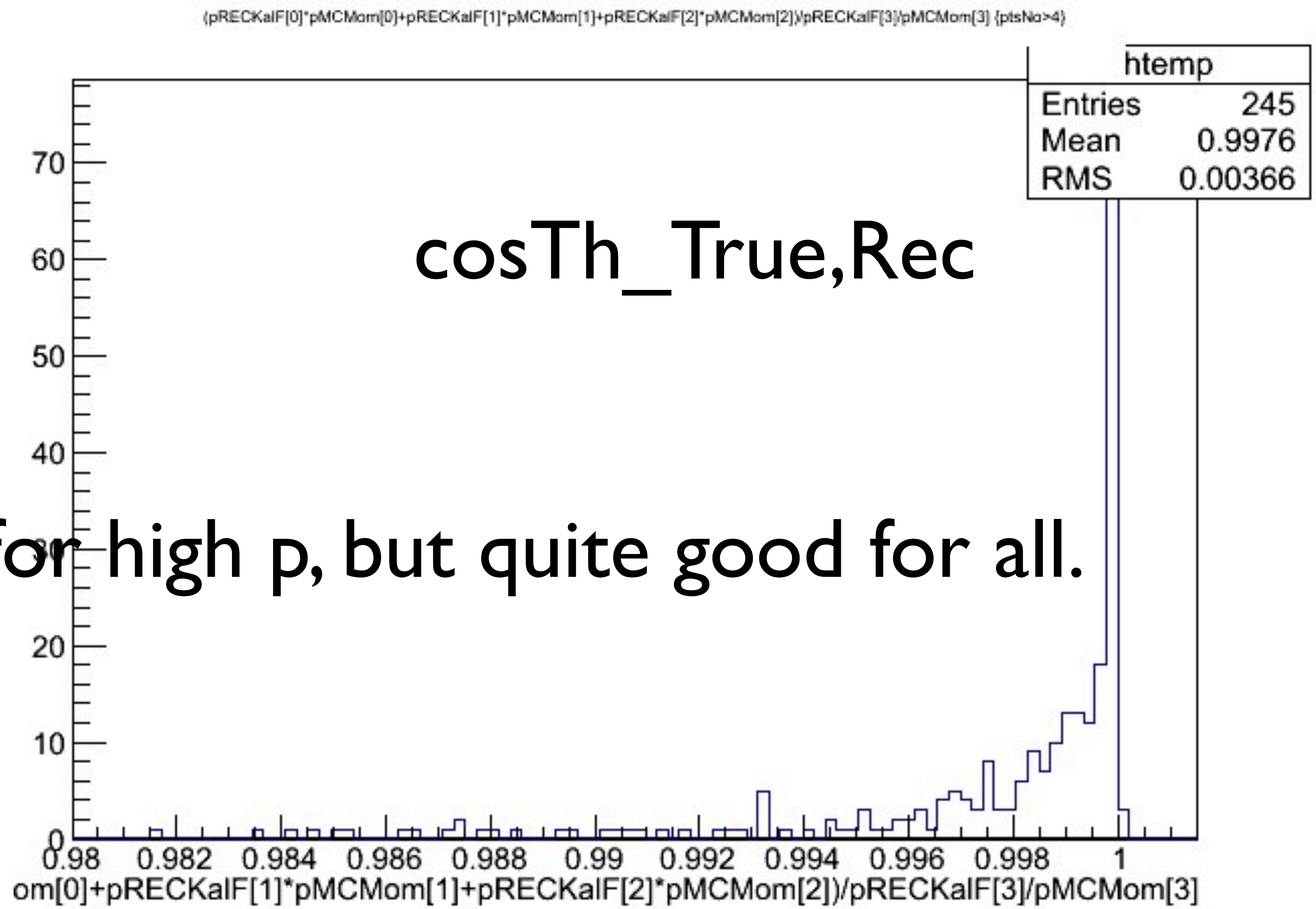
point-to-point scatter angle



I should really show the th weighted by 1/dth, but I don't yet have dth in my TTree.

For this 7 GeV/c track,
MS predicts for 35cm separation
<th>~2.6 mrad

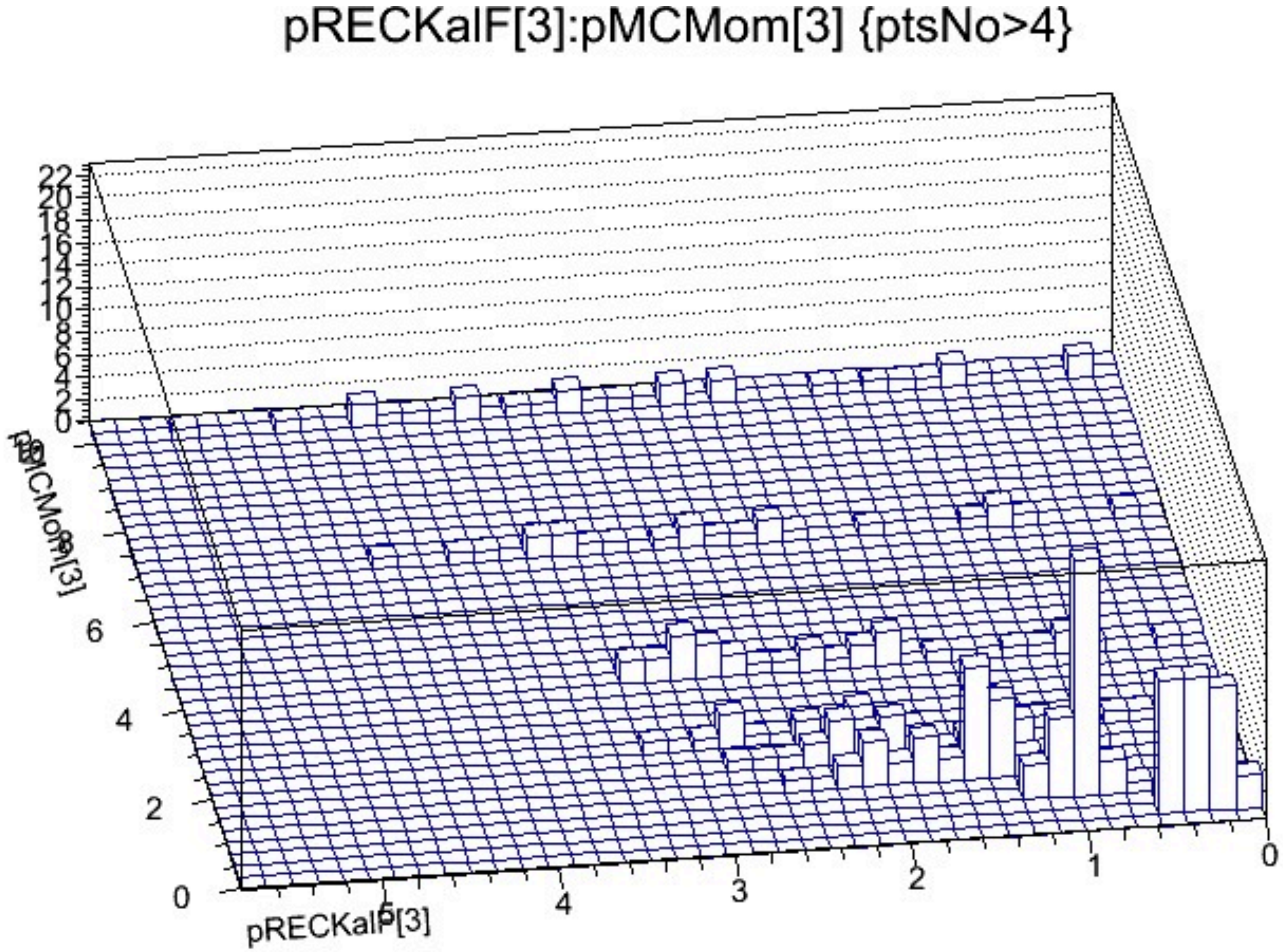
pointing resolution with 100th spacepoint



cosTh_True,Rec

Tighter for high p, but quite good for all.

p resolution for this decimation



Meh.

Example Output after fit.

Initial State:

	0
1/p	0.249
du/dw	0
du/dv	0
u	-0.4686 [cm]
v	-0.4729 [cm]

Never mind that 0.249 should be 0.14!

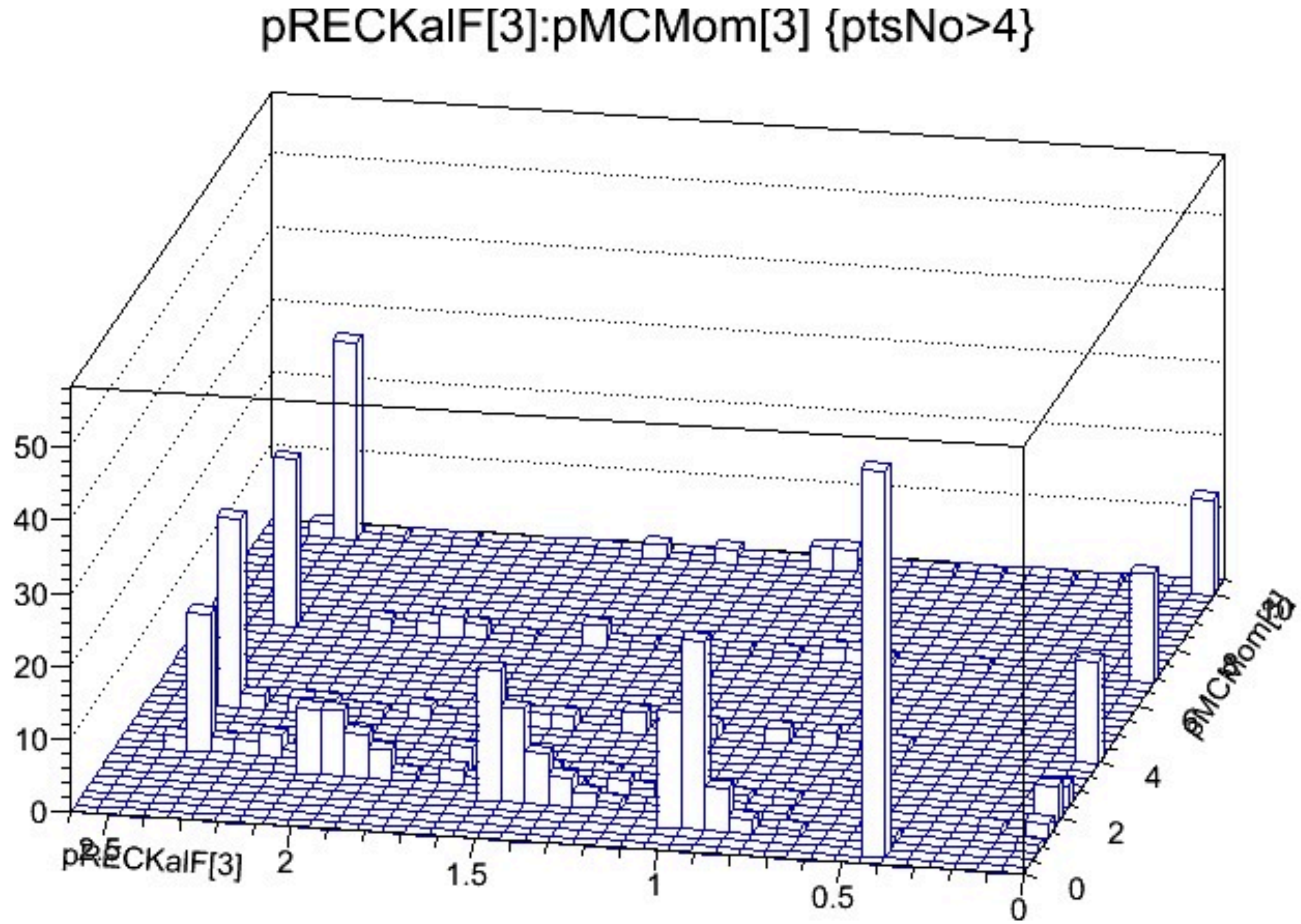
Initial Cov matrix:

5x5 matrix is as follows

	0	1	2	3	4
0	3.239	0	0	0	0
1	0	3.723e-05	2.834e-05	-0.00125	-0.00125
2	0	2.834e-05	3.733e-05	-0.001249	-0.001254
3	0	-0.00125	-0.001249	0.1083	0.1085
4	0	-0.00125	-0.001254	0.1085	0.1087

Depressing realization

- I can force dTheta->inf and this works with ranging out (not contained) muons. Track pointing errors are meaningless.
- Or, simply use 2.2MeV/c*trackLength...
- *Meaning, this is a way to get an initial direction:* Pointing is slightly inferior: rms on cosTh_true,rec goes from 0.003 to 0.06, but mean is still 0.98. Track errors are still meaningless.
- But, just a range-out calculation is undeniably better for contained muons:



Contained tracks with no fit:
~10-15% resolution!

Next

- Don’t quite yet have the off-diagonal Spacepoints errors in, ... Not entirely sure my error implementation is correct everywhere.
- Sort Spacepoints by neighbor, not z. Clean up the code.
- “Average” track parameters over all unique decimations, so as not to waste data.
- As for contained tracks: muons, protons, pions ...

Next, continued

- Then, while still on muons I'd like to try the fit under the pion and/or proton hypothesis. This is as simple as changing the 13 to 211 and/or 2212 in instantiating the track, as below. Then run MC pions/protons and re-fit under these three hypotheses. Just would like to see that the extracted momenta (extracted from the ranging out of these very modestly curved tracks) is different under the three hypotheses

- Then use it on Ornella's/Kinga's/Saima's short tracks in data.

```
rep = new genf::RKTrackRep((TVector3)(spacepoints[0].XYZ()),
                                              momM,
                                              posErr,
                                              momErrFit,
                                              -13);  // mu+ hypothesis
```

Next, again

- I can imagine that 7+ GeV/c tracks are nearly hopeless. *Icarus paper confesses as much.*
- However, non-contained GeV/c tracks should allow this procedure to extract a meaningful p. This will still be useful to uBooNE: a 1.5 GeV/c track gives <th>~10 mrad with spacepoints at every 10 cm for 5m. *I would like to look at these next.*
- I have spent/wasted a lot of time on a category of tracks that exceed our resolution.

Next, yet again

- Would really like to get someone to jump in here. I would like to stay involved, ...
- Lots to do.
- I think even with the 2D Kalman project gaining steam, this method will continue to have application.